USN | | | | | | | | | |

10MAT31

Third Semester B.E. Degree Examination, June/July 2013

Engineering Mathematics - III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain the Fourier series expansion of $f(x) = \begin{cases} x, & \text{if } 0 \le x \le \pi \\ 2\pi - x, & \text{if } \pi \le x \le 2\pi \end{cases}$ and hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots$ **(07 Marks)**

b. Find the half range Fourier sine series of $f(x) = \begin{cases} x, & \text{if } 0 < x < \pi/2 \\ \pi - x, & \text{if } \pi/2 < x < \pi \end{cases}$. **(06 Marks)**

c. Obtain the constant term and coefficients of first cosine and sine terms in the expansion of y from the following table: **(07 Marks)**

x	0	60°	120°	180°	240°	300°	360°
y	7.9	7.2	3.6	0.5	0.9	6.8	7.9

2 a. Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| \le a \\ 0, & |x| > a \end{cases}$ and hence deduce $\int_0^\infty \frac{\sin x - x\cos x}{x^3} dx = \frac{\pi}{4}$. **(07 Marks)**

b. Find the Fourier cosine and sine transform of $f(x) = xe^{-ax}$, where $a > 0$. **(06 Marks)**

c. Find the inverse Fourier transform of e^{-s^2}. **(07 Marks)**

3 a. Obtain the various possible solutions of one dimensional heat equation $u_t = c^2 u_{xx}$ by the method of separation of variables. **(07 Marks)**

b. A tightly stretched string of length ℓ with fixed ends is initially in equilibrium position. It is set to vibrate by giving each point a velocity $V_0 \sin\left(\frac{\pi x}{\ell}\right)$. Find the displacement u(x, t). **(06 Marks)**

c. Solve $u_{xx} + u_{yy} = 0$ given u(x, 0) = 0, u(x, 1) = 0, u(1, y) = 0 and $u(0, y) = u_0$, where u_0 is a constant. **(07 Marks)**

4 a. Using method of least square, fit a curve $y = ax^b$ for the following data. **(07 Marks)**

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

b. Solve the following LPP graphically:
Minimize $Z = 20x + 16y$
Subject to $3x + y \ge 6$, $x + y \ge 4$, $x + 3y \ge 6$ and $x, y \ge 0$. **(06 Marks)**

c. Use simplex method to
Maximize $Z = x + (1.5)y$
Subject to the constraints $x + 2y \le 160$, $3x + 2y \le 240$ and $x, y \ge 0$. **(07 Marks)**

PART – B

5 a. Using Newton-Raphson method find a real root of $x + \log_{10} x = 3.375$ near 2.9, corrected to 3-decimal places. **(07 Marks)**

b. Solve the following system of equations by relaxation method:
$12x + y + z = 31$, $2x + 8y - z = 24$, $3x + 4y + 10z = 58$ **(07 Marks)**

c. Find the largest eigen value and corresponding eigen vector of following matrix A by power method

$$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}.$$

Use $X^{(0)} = [1, 0, 0]^T$ as the initial eigen vector. **(06 Marks)**

6 a. In the given table below, the values of y are consecutive terms of series of which 23.6 is the 6th term, find the first and tenth terms of the series. **(07 Marks)**

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

b. Construct an interpolating polynomial for the data given below using Newton's divided difference formula. **(07 Marks)**

x	2	4	5	6	8	10
f(x)	10	96	196	350	868	1746

c. Evaluate $\int_0^1 \frac{x}{1+x^2}\,dx$ by Weddle's rule taking 7-ordinates and hence find $\log_e 2$. **(06 Marks)**

7 a. Solve the wave equation $u_{tt} = 4u_{xx}$ subject to $u(0, t) = 0$; $u(4, t) = 0$; $u_t(x, 0) = 0$; $u(x, 0) = x(4 - x)$ by taking $h = 1$, $k = 0.5$ upto four steps. **(07 Marks)**

b. Solve numerically the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0, t) = 0 = u(1, t)$, $t \geq 0$ and $u(x, 0) = \sin \pi x$, $0 \leq x \leq 1$. Carryout computations for two levels taking $h = 1/3$ and $k = 1/36$. **(07 Marks)**

c. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown in Fig.Q7(c). **(06 Marks)**

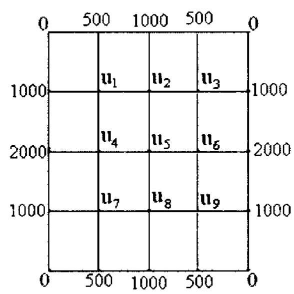

Fig.Q7(c)

8 a. Find the z-transform of: i) $\sinh n\theta$; ii) $\cosh n\theta$. **(07 Marks)**

b. Obtain the inverse z-transform of $\frac{8z^2}{(2z-1)(4z-1)}$. **(07 Marks)**

c. Solve the following difference equation using z-transforms:
$y_{n+2} + 2y_{n+1} + y_n = n$ with $y_0 = y_1 = 0$ **(06 Marks)**

* * * * *